THE General Radio EXPERIMENTER

ELECTRICAL MEASUREMENTS AND THEIR INDUSTRIAL APPLICATIONS

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Also

IN THIS ISSUE

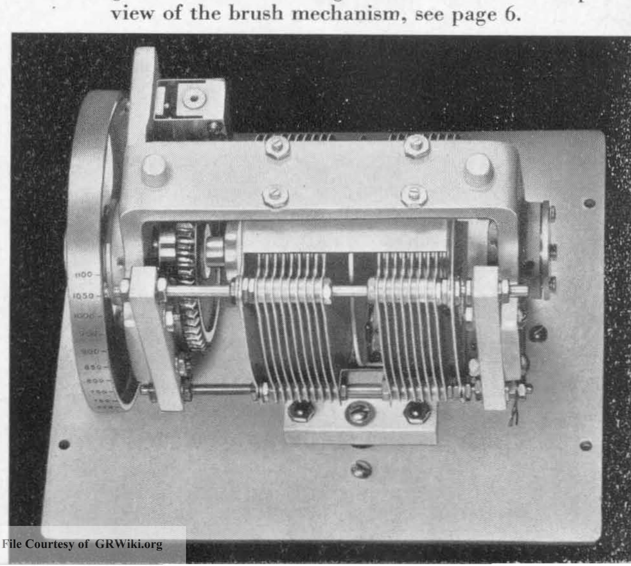
A HIGH-FREQUENCY MODEL OF THE PRECISION CONDENSER

● FOR MANY YEARS General Radio Precision Condensers have been used as basic equipment in laboratories all over the world. The ruggedness, stability and accuracy of these condensers have rendered them of fundamental use in all kinds of measurement work where dependable, continuously-adjustable capacitance standards are required.

The principal features which have led to the widespread adoption of General Radio Precision Condensers are the excellence of the mechanical construction, the precision of capacitance setting, and the low and known electrical losses at audio and low radio frequencies.

In recent years, interest in measurements at high radio frequencies has led to the use of these condensers at frequencies in excess of those for which they were designed. Under these conditions electrical errors arise because of the presence of unwanted residual parameters. Unfortunately the high available precision of capacitance setting in many cases tends to create a feeling of false security and the loss of accuracy in the condenser is not recognized.

FIGURE 1. Interior view of the TYPE 722-N Condenser showing the method of feeding the rotor. For a close-up view of the brush mechanism, see page 6.



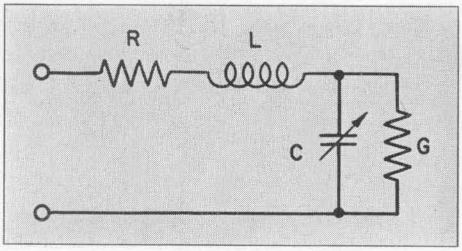


FIGURE 2.
In this circuit the resistance, R, corresponds to losses in the metallic portions of the condenser; the conductance, G, corresponds to losses in the solid dielectric portions of the condenser; and the inductance, L, corresponds to magnetic flux set up by conduction currents in the metal portions of the condenser. The capacitance, C, represents the static capacitance of the condenser.

The addition to the Precision Condenser line of a new high-frequency model, the TYPE 722-N, extends the advantages of highly precise mechanical construction to a condenser whose performance can be accurately predicted at frequencies up to 30 Mc.

RESIDUAL PARAMETERS

The residual electrical parameters which occur in variable air condensers and which cause the behavior to change as a function of frequency are: (1) resistance components corresponding to losses in the metal and solid dielectric portions of the condenser, and (2) inductance caused by the magnetic field set up by conduction currents in the metal structure.

An equivalent circuit which may be used to represent a variable air condenser is shown in Figure 2*.

As a function of dial setting the residual parameters designated by R, G, and L all tend to remain constant. As a function of frequency the inductance, L, remains constant, the conductance, G, increases nearly linearly with frequency and, at high frequencies where it is significant, the resistance, R, increases approximately as the square root of the frequency.

*R. F. Field and D. B. Sinclair, "A Method for Determining the Residual Inductance and Resistance of a Variable Air Condenser at Radio Frequencies," *Proc. I. R. E.*, 24, 2, February, 1936.

EFFECTS OF RESIDUAL PARAMETERS

The residual inductance, L, introduces a component of positive reactance in series with the condenser, which causes the net negative reactance at the terminals to be lower than it should be. The effect of the inductance is therefore to increase the terminal capacitance by a fractional amount which increases as the capacitive reactance decreases and as the inductive reactance increases. The error consequently increases both with frequency and with dial setting. The effective terminal capacitance follows the law

$$C_e \simeq \frac{C}{1 - \omega^2 LC} \qquad (1)$$

The conductance, G, causes a dissipative component in the terminal impedance.

Since the conductance, G, increases linearly with frequency, the corresponding component of dissipation factor

$$D_G = \frac{G}{\omega C}$$

is constant as a function of frequency at any given capacitance setting.

The resistance, R, adds a further dissipative component of terminal impedance.

The corresponding dissipation factor component

$$D_R = R\omega C$$

is ordinarily negligible up to frequencies at which skin-effect in the metal parts is essentially complete. At higher frequencies the resistance, R, increases as the square root of the frequency and the dissipation factor component increases as the three-halves power of the frequency.

A precision condenser is used normally under such conditions that the dissipation factor components, D_G and D_R, and the inductive error are small. The expressions for the effective terminal impedance and admittance of the condenser under these conditions are

$$Z_e = R_e - j\frac{1}{\omega C_e}$$

$$\simeq \left[R + \frac{G}{(\omega C)^2}\right] - j\left[\frac{1 - \omega^2 LC}{\omega C}\right] \quad (2)$$

$$Y_e = G_e + j\omega C_e$$

$$\simeq \left[G + R(\omega C)^2\right] + j\left[\frac{\omega C}{1 - \omega^2 LC}\right] \quad (3)$$

and the over-all dissipation factor is approximately

$$D = D_G + D_R = \frac{G}{\omega C} + R\omega C \quad (4)$$

ERRORS IN MEASUREMENTS CAUSED BY RESIDUAL PARAMETERS

The errors caused by residual parameters in measurements using a variable air condenser as standard depend upon the frequency and upon the method of measurement. At high frequencies, in particular, it has been found that substitution methods of measurement tend to give results of maximum accuracy. In this discussion the parallel-substitution method will be the only method considered.

In parallel substitution methods the susceptance of a given circuit branch containing the standard condenser is set at some particular value corresponding to a desirable capacitance setting. The unknown admittance is then connected in parallel with the standard condenser and the susceptance restored to its initial value by readjusting the condenser. The susceptive component of the unknown is found directly from the change in susceptance of the condenser. The conductive component of the unknown is found from the change in total conductance of the arm when the unknown admittance is in and out of circuit.

Measurement errors can arise from three sources if the residual parameters of the condenser are neglected:

(1) The change in susceptance of the standard condenser between the initial and final condenser readings is not equal to $\omega(C_2 - C_1)$ but is influenced by the residual inductance and is equal to

$$\omega(C_{e_2} - C_{e_1}) = \frac{\omega(C_2 - C_1)}{1 - \omega^2 L(C_1 + C_2)} \quad (5)$$

(2) The conductance of the standard condenser does not remain constant but changes between the initial and final settings by an amount

$$G_2 - G_1 = R\omega^2 C_x(C_{e_1} + C_{e_2}) \quad (6)$$

(3) If parallel-resonance methods, such as the susceptance-variation method,* are used to determine the dissipative component of the unknown, the observed breadth of the resonance curve is influenced by residual inductance. For the breadth of the resonance curve used to determine conductance, the true capacitance difference to be used is

$$\triangle C_e = \frac{C'' - C'}{1 - \omega^2 L(C' + C'')} \quad (7)$$

where C' and C'' are the two readings on either side of resonance.

The effect of residual parameters is greatest in the measurement of small values of power factor such as those of good mica condensers. An example of

*D. B. Sinclair, "Parallel Resonance Methods for Precise Measurements of High Impedances at Radio Frequencies," *Proc. I. R. E.*, December, 1938.

the large errors which may be encountered under extreme conditions is as follows:

The inductance of the 1000 $\mu\mu$f section of a Type 722-D Precision Condenser is approximately 0.065 μh and the metallic resistance at a frequency of 10 Mc is about 0.065 Ω. Suppose this condenser be used to measure the capacitance and power factor of a 1000 $\mu\mu$f Type 505 Condenser at a frequency of 10 Mc.

The effective capacitance of the 1000 $\mu\mu$f Type 505 Condenser at 10 Mc is 1258 $\mu\mu$f and the power factor is 0.9%.[1] Let the initial dial reading of the standard condenser, C_1, be 1100 $\mu\mu$f. The initial effective terminal capacitance is

$$C_{e_1} = \frac{C_1}{1 - \omega^2 LC_1} = 1532\ \mu\mu\text{f}$$

The final effective terminal capacitance must be

$$C_{e_2} = 1532 - 1258 = 274\ \mu\mu\text{f}$$

and the final dial reading

$$C_2 = 254\ \mu\mu\text{f}.$$

The error in taking the difference in dial readings as the unknown capacitance, without correction for inductance, is therefore

$$1 - \frac{1100 - 254}{1258} \times 100 = 32.8\%$$

The component of condenser conductance caused by metallic losses at the initial setting is

$$R(\omega C_{e_1})^2 = 602\ \mu\text{mho}$$

[1]The effective capacitance is greater than the nominal capacitance because of inductance. See "The Behavior of Type 505 Condensers at High Frequencies," General Radio *Experimenter*, April, 1938.

and at the final setting

$$R(\omega C_{e_2})^2 = 19\ \mu\text{mho}$$

The change in condenser conductance is therefore −583 μmho when the susceptance is restored after connecting the unknown. The conductance of the 1000 $\mu\mu$f condenser corresponding to a power factor of 0.9% is 867 μmho. The error in taking as the conductance of the unknown the difference in conductance of the circuit when the unknown is connected and disconnected is therefore

$$\frac{583}{867} \times 100 = 67.2\%$$

Very large errors in both capacitance and power-factor measurements are seen to occur. Indeed, in many cases the error caused by metallic resistance is so large as to cause the observed value of power factor to become negative.

LOCATION AND REDUCTION OF RESIDUALS IN TYPE 722-N PRECISION CONDENSER

The minimization of the residual inductance and metallic resistance is seen to be a prime requisite in the design of a high-frequency condenser.

The residual resistance arises in the rotor shaft and stator rod washers, in the washer-to-plate contacts, and in the plates themselves.[2] The residual inductance arises principally from magnetic flux set up by currents in the rotor shaft and stator rod washers. This flux lies in planes parallel to the plates. Currents in the plates themselves set up relatively little flux since they are diffused over large areas.

[2]At high frequencies the current tends to the path of least inductance which is around the plates, rather than through them. The losses in the plates therefore become an appreciable part of the whole. The reason that the metallic resistance remains relatively constant with dial setting is apparently found in the fact that the major loss occurs in the immediate vicinity of the rotor shaft and stator rods where the current density is high. In these regions the current distribution is not so greatly affected by rotor position as elsewhere.

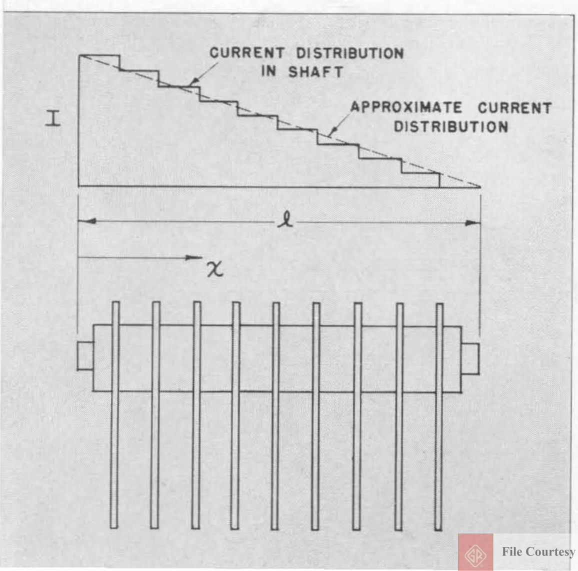


Figure 3. Showing the distribution of current in a rotor shaft fed at the left-hand end.

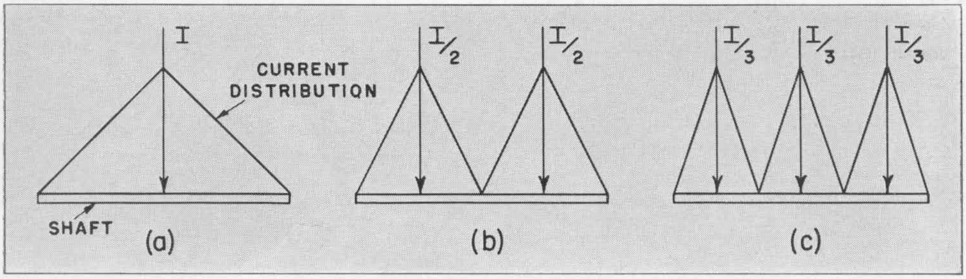


FIGURE 4. Current distribution when current is fed symmetrically to the shaft.

To a very fair degree of approximation the metallic resistance and residual inductance of a variable air condenser can be considered as uniformly distributed along the rotor shaft and stator rods. On this basis a simple analysis of the effect of points of current entry into the stack can be formulated.

Figure 3 illustrates a rotor shaft with current fed in at the left-hand end. To a first approximation the current decreases linearly along the shaft length at frequencies low compared to the first natural frequency.

Suppose the resistance of the shaft to uniform current is R and the inductance L. The effective resistance and inductance for the non-uniform current are easily found from energy considerations.

The current at any distance along the shaft, i, is related to the current at the left-hand end, I, by the expression

$$i = I\frac{l - x}{l}$$

The total power loss, referred to the left-hand end of the shaft, is

$$I^2R_e = \int_0^l i^2 \frac{R}{l} dx$$
$$= \frac{R}{l}\frac{I^2}{l^2}\int_0^l (l - x)^2\, dx = I^2\frac{R}{3}$$

and the effective resistance $R_e = R/3$.

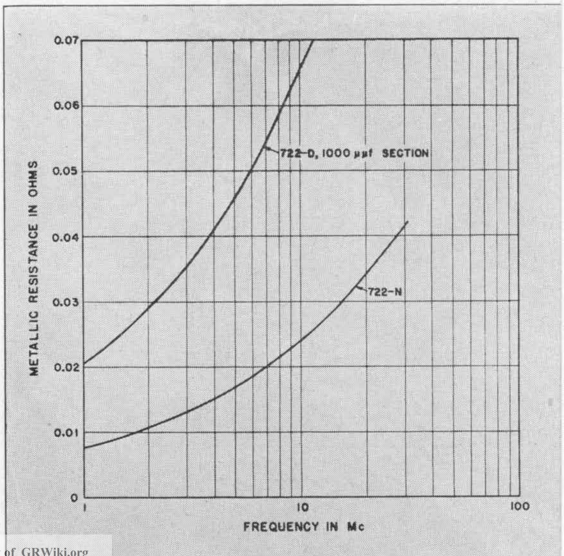
Similarly the total energy storage, referred to the left-hand end of the shaft, is

$$\frac{1}{2}L_eI^2 = \frac{1}{2}\int_0^l \frac{L}{l} i^2\, dx$$
$$= \frac{1}{2}\frac{L}{l}\frac{I^2}{l^2}\int_0^l (l - x)^2\, dx$$
$$= \frac{1}{2}\left(\frac{L}{3}\right)I^2$$

and the effective inductance $L_e = L/3$.

The effective resistance and inductance can be reduced by feeding current symmetrically to the shaft. For instance, if the current be fed at the center instead of the end the current distribution is as

FIGURE 5. Metallic resistance of Type 722-N Precision Condenser as a function of frequency. For purposes of comparison, the resistance of TYPE 722-D is also shown.



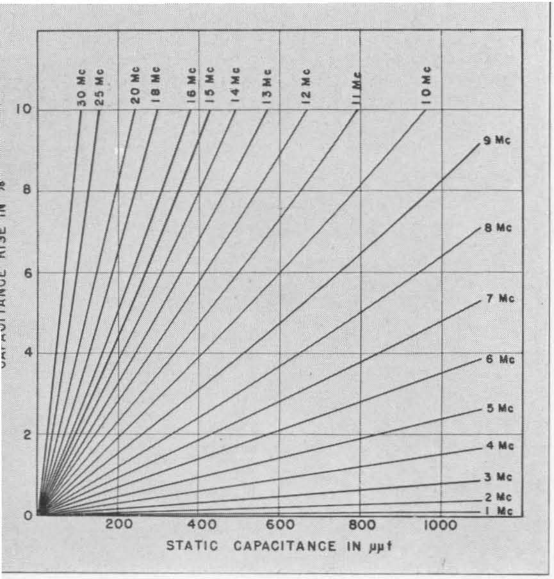
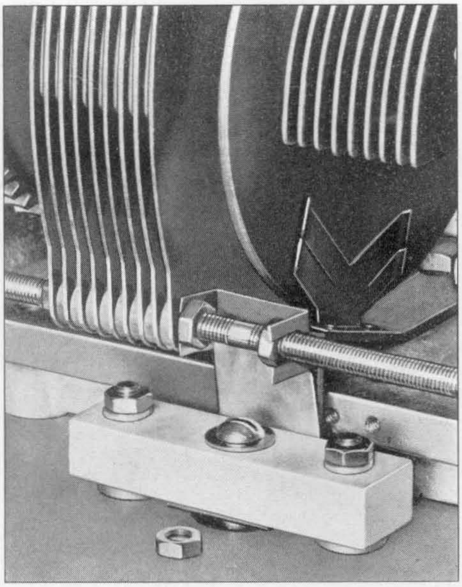


FIGURE 6. Variation in effective capacitance of TYPE 722-N Condenser as a function of static capacitance for various frequencies.

shown in Figure 4(a) and the effective resistance is $R_e = R/12$ and the effective inductance $L_e = L/12$.

Multiple current feed reduces the residual parameters still further. Double feed, as in Figure 4(b), gives $R_e = R/48$ and $L_e = L/48$; triple feed, as in Figure 4(c), gives $R_e = R/108$ and $L_e = L/108$.

FIGURE 7. Showing the leads and the method of connection to the rotor.



The general expression for n points of entry into the stack is $R_e = R/12\,n^2$ and $L_e = L/12\,n^2$.

PRACTICAL APPLICATION OF SYMMETRICAL FEED TO CONDENSER

Change-over from the usual end-feed system to a center-feed system lowers both the metallic resistance and residual inductance by a factor of 4. In practice, it is seldom advantageous to go further than this because the resistance and inductance of the leads to the binding posts quickly become predominant.

In the TYPE 722-N Precision Condenser center-feed has been adopted with a consequent reduction of resistance and inductance in the stack. In addition a heavy strip connector is used to feed the stator stack and a brass disc with a wide brush contactor to feed the rotor. A detailed view of the construction is shown in the accompanying photograph.

The metallic resistance and residual inductance obtained with this construction are lower by a factor of about 3 : 1 than those obtained with the high section of the TYPE 722-D Precision Condenser. For a typical TYPE 722-N Precision Condenser the variation of the metallic resistance with frequency is shown in Figure 5. The residual inductance is constant and is equal to 0.024 μh. The variation in effective terminal capacitance caused by this inductance is illustrated in Figure 6.

Because an insulated rotor shaft is used, no current flows in the ball bearings which support the rotor shaft. This construction prevents the variation of metallic resistance which would otherwise arise in the erratic electrical contacts between the bearing surfaces.

— D. B. SINCLAIR

SPECIFICATIONS

Capacitance Range: 100 to 1100 $\mu\mu$f, direct reading.

Rotor Plate Shape: Semicircular to give a linear capacitance characteristic.

Standard-Calibration Accuracy: The capacitance, measured at 1000 cycles, is indicated directly in micromicrofarads by the dial and drum readings to ± 1 $\mu\mu$f.

Worm-Correction Calibration: A worm correction can be supplied on special order. (See price list.) A mounted chart is supplied giving the correction to at least one more figure than the guaranteed accuracy stated below.

When this correction is used, the capacitance can be determined within ± 0.1 $\mu\mu$f or $\pm 0.1\%$, whichever is the greater, and capacitance differences can be measured to an accuracy of ± 0.2 $\mu\mu$f or $\pm 0.1\%$, whichever is the greater.

Dielectric Supports: Two bars of isolantite support the stator assembly, and a third insulates the high terminal from the panel.

Dielectric Losses: The figure of merit, $R\omega C^2$, when measured at 1000 cycles, is approximately 0.05×10^{-12}.

Other Residual Parameters: See Figures 5 and 6.

Maximum Voltage: 1000 volts, peak.

Temperature Coefficient: Approximately $+0.002\%$ per degree Centigrade.

Mounting: The condenser is mounted on an aluminum panel finished in black crackle lacquer and enclosed in a shielded walnut cabinet. A wooden storage case with lock and carrying handle is included.

Dimensions: Panel, 8 x 9⅛ inches; depth, 8⅛ inches.

Net Weight: 11⅛ pounds; 20¼ pounds with carrying case.

Type	*Description*	*Code Word*	*Price*
722-N	100 to 1100 $\mu\mu$f, direct reading	BOXER	$150.00
Worm-Correction Calibration		WORMY	35.00

When ordering use compound code word, BOXERWORMY.

THE PRECISION FORK IN CONTINUOUS OPERATION

● **THE TYPE 815-A PRECISION FORK** announced in the May, 1936, issue of the *Experimenter* has been widely used as a secondary standard of frequency for standardization and measurement where a precision of one part in ten thousand (0.01%) is adequate. A considerable number have been used as the timing elements in seismographic surveying for oil deposits, as reliably steady sources of alternating current for the stroboscopic regulation of clocks and watches, and as the synchronizing elements in facsimile transmission, etc. They afford a simple means of providing stabilized alternating current in the low audible frequency range without the elaborate equipment required to produce these low-frequency currents from a piezo-electric oscillator.

These forks are constructed of a special stainless steel alloy which gives them a much lower temperature coefficient of frequency (less than ten parts per million negative per degree F.) than ordinary machine steel, so that frequently they are used without temperature control, and their design is such that the voltage coefficient of the driving battery (which for intermittent operation may be simply three dry cells) is quite negligible. They are readily portable and can be made for any frequency between 40 and 200 cycles per second.

The fork is massive, accurately machined, and mounted on rubber shock absorbers. Two microphone buttons are used, one for driving the fork, the other to supply energy at the fork frequency to an external circuit.

The author recently had occasion to investigate how one of these forks would behave on continuous operation under admittedly ideal conditions.

In order to eliminate the small effect of temperature fluctuations, a 50-cycle