

A CLOSE LOOK AT CONNECTION ERRORS IN CAPACITANCE MEASUREMENTS

The growing interest in capacitance measurements both of higher accuracy and of smaller capacitance has led to a re-examination of some of the problems involved in the *precise* and *accurate* measurement of small capacitance. It has been evident for some time that errors and uncertainties of the order of a few tenths of a picofarad were present in most measurements of two-terminal capacitors.¹ Such errors are not very significant in the calibration of standard capacitors as long as the capacitance exceeds 100 pf and the desired accuracy is no greater than 0.1%. Any attempt however, to calibrate smaller capacitors to this accuracy, or to increase the accuracy of other calibrations, demands a consideration of the accuracy limitations imposed by the connection errors in the usual two-terminal measurements.

¹The common capacitor is here characterized specifically as two-terminal because a three-terminal capacitor will be introduced and defined later.

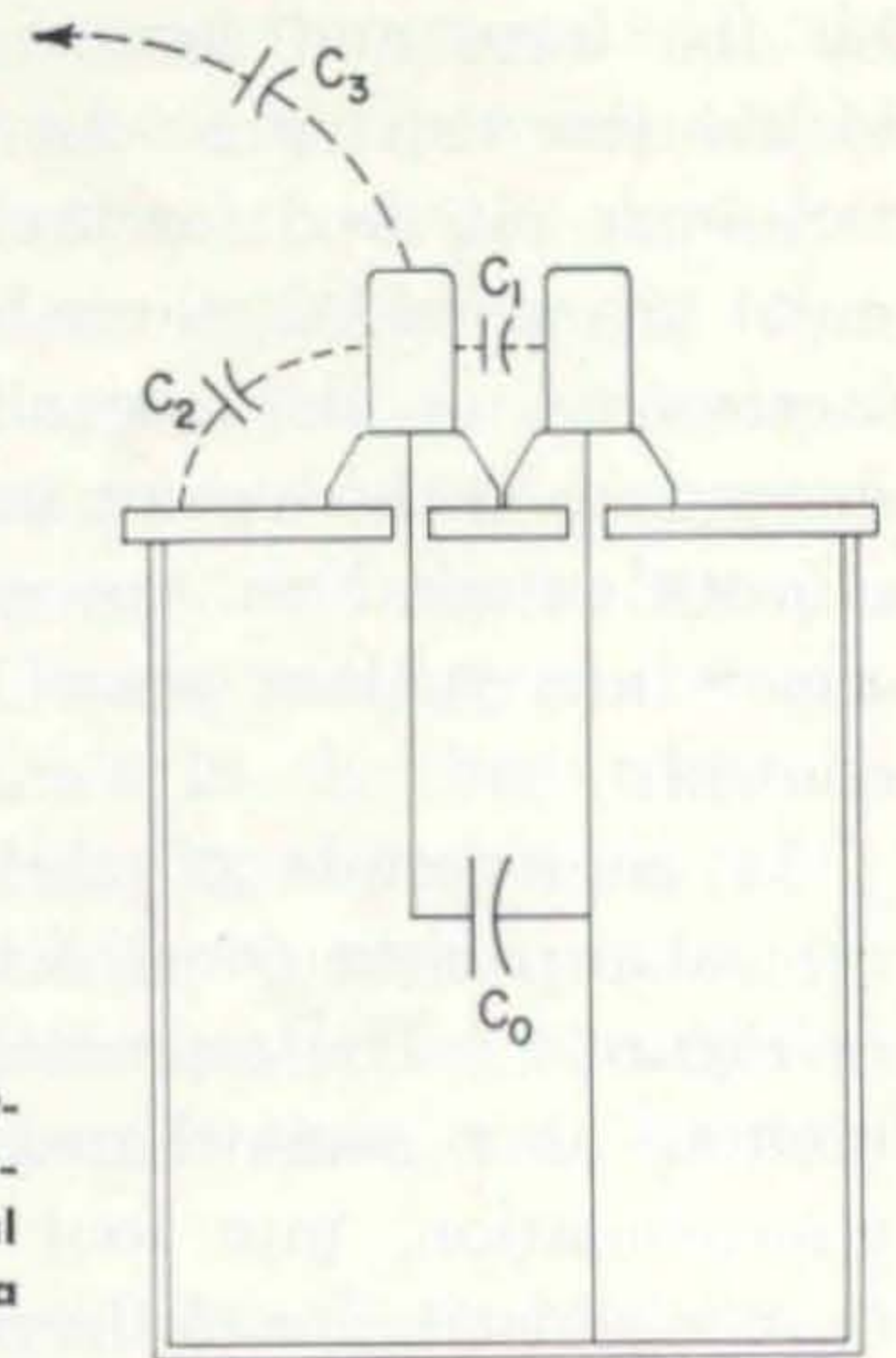
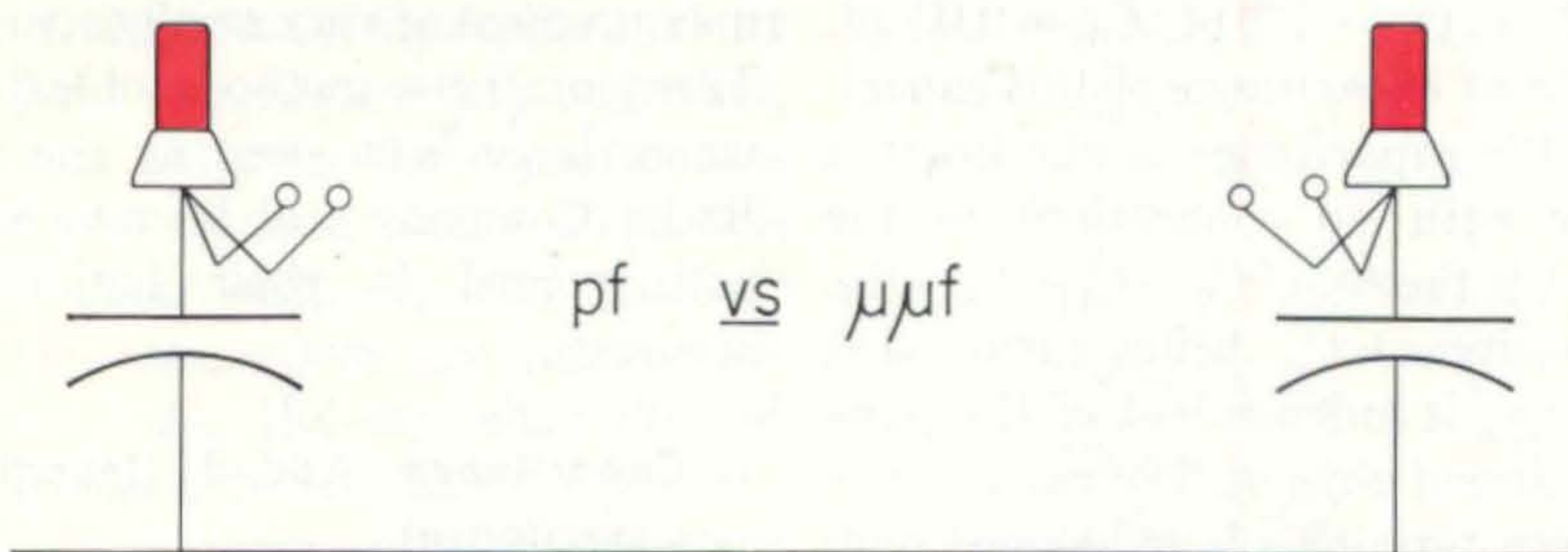


Figure 1. Schematic diagram showing the terminal capacitances of a capacitor.

The problems arise from the connections that must be made to a capacitor in order either to use or to measure its capacitance. The capacitance is, of course, determined by the geometrical configuration of the conductors (and by the dielectric material, which will here



As our capacitance measurements and standards move toward the millionth part of the millionth part of the millionth part of the farad (10^{-18} f), we have found increasing advantage in following the lead of the National Bureau of Standards and others in calling 10^{-12} farad a *picofarad* instead of a micromicrofarad. A privilege most prized by proponents of the picofarad is the right to write the abbreviated abbreviations 1 pf instead of 1 $\mu\mu\text{f}$ for 10^{-12} farad and 1 μpf instead of 1 $\mu\mu\mu\text{f}$ for 10^{-18} farad.

be assumed to have simple and constant characteristics). Only when one conductor completely surrounds the other is the capacitance simply defined by the form and nature of materials inside the capacitor. As soon as the terminals required for use or measurement are provided, these terminals add increments to the capacitance, which depend upon the nature and position of objects external to the capacitor and which are seldom easy to define or control.

As an example of this, consider a typical capacitor constructed as shown in Figure 1.² The capacitance has been broken, as a somewhat arbitrary first approximation, into four components: C_0 , the capacitance of the multiple-plate capacitor and leads within the case; C_1 , the capacitance between the external binding posts; C_2 , the capacitance between the high terminal and the case, which is connected to the other terminal; C_3 , the capacitance between the high terminal and all objects external to the capacitor and its terminals. Typical of the magnitudes of these components are the values $C_0 = 100$ pf, $C_1 = 0.2$ pf, $C_2 = 1.3$ pf, $C_3 = 0.03$ pf.

The "free" capacitance of this capacitor, i.e., the capacitance of the isolated capacitor with no connections to the terminals, is the sum,³ $C_0 + C_1 + C_2 + C_3$. The capacitance C_0 , being surrounded by the case, is independent of the position of external objects. The capacitance C_1 between terminals is influenced only by intrusions very close to or between the terminals. But C_3 , and to a lesser

extent C_2 , can readily be affected by more distant environment of the capacitors, and variations of 0.01 pf or more can result from a slight change of position.

More radical changes in these external capacitances are produced by any connections made to the terminals. A wire connected to the high terminal, for example, obviously introduces new components of capacitance between the wire and the capacitor parts. It also, not so obviously, reduces the "free" capacitance by as much as 0.1 pf by changing the distribution of field around the terminals. There is a similar reaction of the capacitor on the connections, which makes the capacitance of the leads when connected to the capacitor differ from that of the leads alone. Indeed, the complexity of this mutual interaction is such that it is impractical to define the dividing line between capacitor and leads with a precision much better than ± 1 pf.

A consideration of these difficulties suggests several possible methods of measurement in which the errors and uncertainties can be reduced or eliminated in order to obtain accuracy in the measurement of very small capacitances. Three of these methods of calibration which have been used at the General Radio Company will be reviewed here with regard to their limitations in accuracy.

1. Capacitance Added (Insertion Capacitance).

Since the capacitance of the two-terminal capacitor depends upon the environment of the capacitor and upon the method of connection, an accurate calibration can be made only by defining with sufficient precision the geometry of both the environment and the connections. One practical method of achieving

²This is the structure used until recently in the General Radio TYPE 1401 Standard Air Capacitors.

³When the capacitor is isolated, C_3 is the capacitance to infinity. The similar capacitance from the low terminal to infinity is in series with C_3 in the "free" capacitance, but it can be neglected here because this capacitance to infinity is much larger than C_3 when the low terminal is connected to the capacitor case and is infinite when the low terminal is grounded, as it usually is in two-terminal measurements.

high accuracy is to calibrate in terms of the capacitance change at a pair of terminals when some change is made in the capacitor or its position. Two measurements are required to determine the capacitance change, and, hence, terminal conditions for both measurements must be either invariant or precisely specified. When the capacitor has variable capacitance, the shielded internal capacitance can be varied in an environment determined solely by the capacitor construction; while the external capacitances at the terminals, although dependent upon external connections and environment, can easily be held so constant during the capacitance change that they make no appreciable contribution to the difference. Such variable capacitors can be calibrated⁴ in terms of the capacitance added or removed by rotation of the capacitor plates with essentially no limitation of accuracy by connection errors.

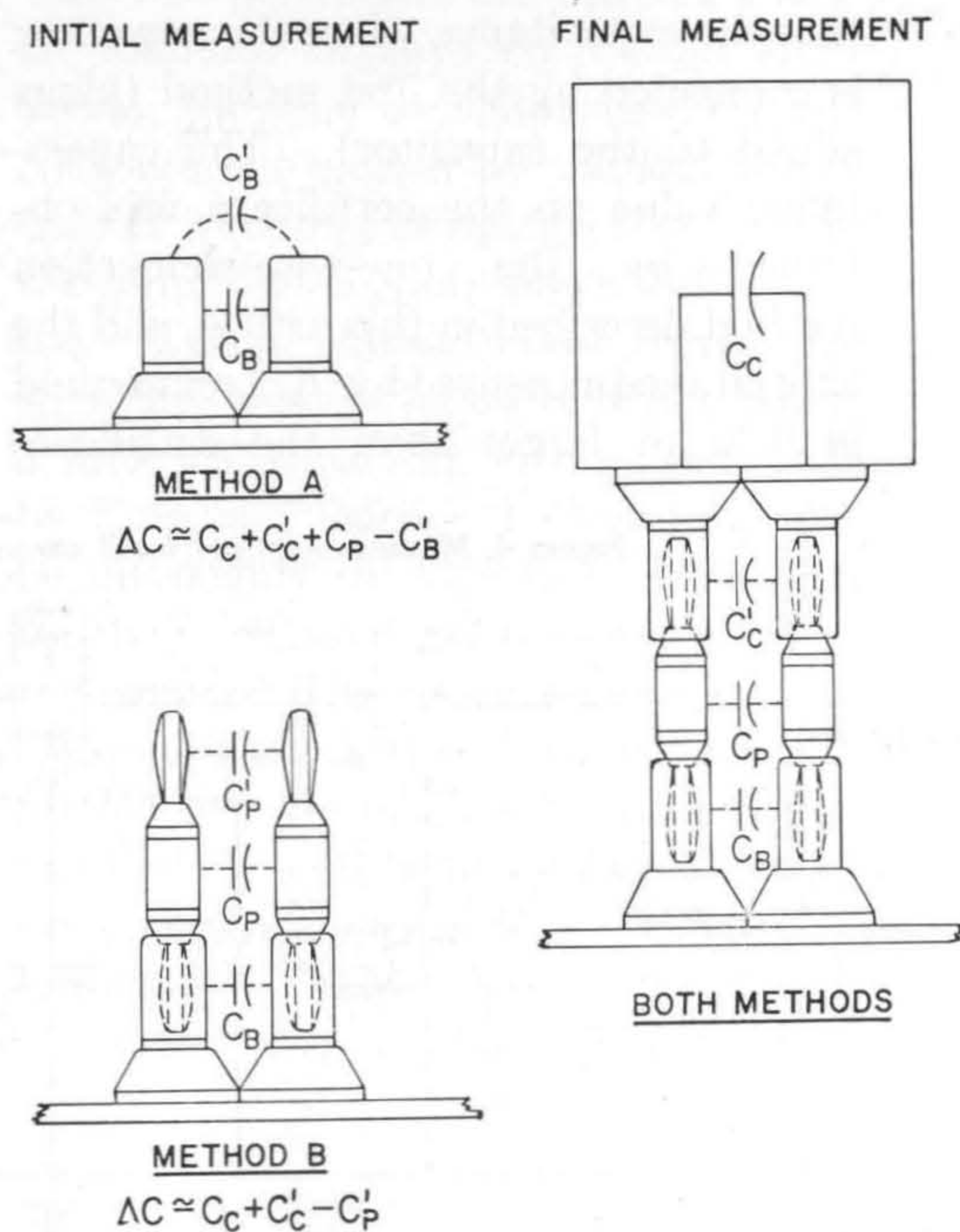
When the capacitor has a fixed capacitance, a calibration of high accuracy of the capacitance added requires that the capacitor be connected with specified leads to a specified set of terminals. For example, the General Radio TYPE 1409 Standard Capacitors are calibrated in terms of the capacitance they add when the banana plugs on the capacitor are plugged into General Radio TYPE 938 Binding Posts with $\frac{3}{4}$ -inch spacing. When the connections are made with reasonable care, the reproducibility of measurement is better than 0.1 pf.

For greater accuracy the environment of the terminals must also be defined, for any change in terminal position or panel size (which results, for example, from the use of different bridges or of terminals on an external capacitor instead of the bridge terminals) can pro-

duce a change in the measured capacitance. The value assigned to the capacitor is the difference between two bridge measurements: the first with the bridge terminals open, and the second with the capacitor connected to the terminals. In the second measurement the capacitor case and bridge panel are usually effective in shielding the bridge terminals, so that this measurement is not very sensitive to changes outside the radius of a few inches. The capacitance of the open bridge terminals in the first measurement (and of any open terminals on the capacitor, such as those on top of the 1409 Capacitors) is affected to a greater extent by panel size and terminal position.

The General Radio TYPE 1401 Standard Air Capacitor is another example of a fixed capacitor calibrated in terms of the added or "insertion" capacitance. These capacitors are now being made, as shown in Figure 3, with banana-plug

Figure 2. Two methods for the measurement of capacitance added. The capacitances shown represent only major components in the complete expression of the self and mutual capacitances of the terminals.



⁴The General Radio TYPE 722-MD and TYPE 722-ME Precision Capacitors are calibrated in terms of capacitance removed.

terminals on the case, similar to those on the TYPE 1409 Capacitors. In previous production the TYPE 1401 Capacitors have had the jack-top TYPE 938 Binding Posts on their cases, and two double-ended banana plugs have been provided with each capacitor to connect it to another pair of similar binding posts. With these connectors two different methods of added-capacitance calibration can be made. In the first method, shown in Figure 2-A, the initial measurement is made with the bridge terminals open and the final measurement with the capacitor connected by means of the double-ended plugs. In the second method, shown in Figure 2-B, the initial measurement is made with the plugs in the bridge terminals and the final with the capacitor added to the plugs.

This choice of methods has in the past resulted in some confusion. The National Bureau of Standards has used the second method (plugs added to the bridge) in its calibrations of these capacitors, but the General Radio Company has given on its calibration certificates a correction of the certificate value to obtain the capacitance when the capacitor is connected by the first method (plugs added to the capacitor). The capacitance value on the certificate was obtained by the fine-wire-connection method described in this article, and the capacitance measured by the first method is 0.35 pf larger than the certificate



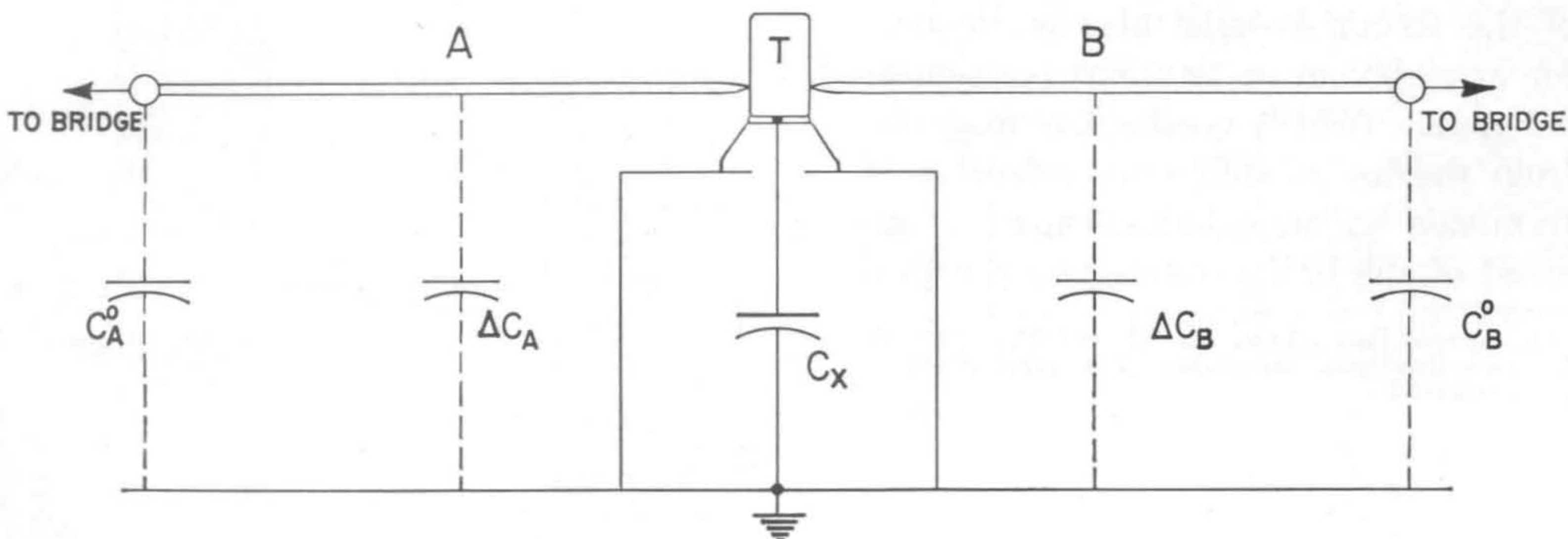
Figure 3. Type 1401 Fixed Air Capacitor as now supplied.

value. When the second method is used, the measured capacitance is 0.7 pf lower than that of the first method because the capacitance added to the bridge terminals by the plugs in the initial measurement ($C_p + C'_p - C'_B$ in Figure 2) is 0.7 pf. In neither method is the added capacitance the same as the "free" capacitance.⁵ In the first method the added capacitance is greater than the "free" capacitance because of the capacitance, C_p , added by the plugs. In the second method the added capacitance is less than the "free" capacitance because the connection of the capacitor removes from the final measurement the capacitance, C'_p , between the ends of the plugs that was included in the initial measurement.

To avoid both this choice between connection methods and also the limitations of the fine-wire-connection method,

⁵The value on the certificate, obtained by the fine-wire-connection method, also differs by about 0.2 pf from the "free" capacitance for reasons explained in this article.

Figure 4. Measurement of "free" capacitance by the Rosa and Dorsey method.





the 1401 Capacitors are now provided with only banana-plug terminals, and they are calibrated by both the National Bureau of Standards and General Radio Company, as the 1409 Capacitors are, in terms of the capacitance added when the capacitor is plugged directly into General Radio type binding posts.

2. "Free" Capacitance.

In the attempt to define a capacitance with high accuracy, an alternative to the method of specifying with sufficient precision the geometry of the connections is a method which eliminates all connections and defines the "free" capacitance of the capacitor with all disturbing connections and surroundings removed. In this method the difficulties of measuring the capacitance of a capacitor, isolated from its surroundings and without connections, are substituted for the difficulties of controlling the geometry of the connections and the environment.

It is possible to determine this "free" capacitance by evaluation of the disturbing effects of the connections and application of a correction for these effects to the measured capacitance. A method for doing this has been described by Rosa and Dorsey.⁶ The effects of the connections can be eliminated from the measurement by the use of two sets of connecting leads which are not necessarily identical but which have no mutual interaction. As a simplified illustration of this method, consider the arrangement shown in Figure 4. The capacitor to be measured has a "free" capacitance C_x , which includes external components corresponding to C_1 , C_2 , C_3 in Figure 1. For simplicity only the high terminal, T , is shown in Figure 4, but the ground connection to the case and to one side of

the capacitor shown in the figure can be made to a second, similar terminal on the capacitor. A first measurement is made with connecting wire A connected to the terminal and to the bridge and with wire B removed. The measured capacitance is $C_1 = C_A^\circ + C_B^\circ + \Delta C_A + C_X - \Delta C_X^A$. A similar measurement with wire B connected and A removed gives $C_2 = C_A^\circ + C_B^\circ + \Delta C_B + C_X - \Delta C_X^B$, where

$C_A^\circ + C_B^\circ$ is the measured capacitance of leads, bridge, etc., when both wires A and B are removed,

ΔC_A and ΔC_B are the increments added by wires A and B ,

ΔC_X^A and ΔC_X^B are the changes in C_X resulting from the presence of A and of B .

With both wires A and B connected to the terminal the capacitance measured is $C_{12} = C_A^\circ + C_B^\circ + \Delta C_A + \Delta C_B + C_X - \Delta C_X^A - \Delta C_X^B$. These relations can be combined to show that $C_X = C_1 + C_2 - C_{12} - (C_A^\circ + C_B^\circ)$. The "free" capacitance C_X can thus be determined from four measured capacitances, C_1 , C_2 , C_{12} , and $C_A^\circ + C_B^\circ$.

In this derivation the wires A and B are assumed to have no mutual interaction, e.g., the capacitances ΔC_A and ΔC_X^A are not altered by the addition of wire B when C_{12} is being measured. In the simple connection shown in Figure 4 there is some mutual effect (which can with care be kept below 0.01 pf), but in a spherical capacitor such as that used by Rosa and Dorsey the two leads can be thoroughly shielded from each other by their locations on opposite sides of the sphere. The assumption that the calculated C_X is the "free" capacitance further requires that the initial measurement of $C_A^\circ + C_B^\circ$ be made with all measuring apparatus so far removed from the terminal T that it has no significant effect on the capacitance. In practice it

⁶E. B. Rosa and N. E. Dorsey, "A New Determination of the Ratio of the Electromagnetic to the Electrostatic Unit of Electricity," Bull. Bureau of Stand., Vol. 3, Nos. 3 and 4, 1907.

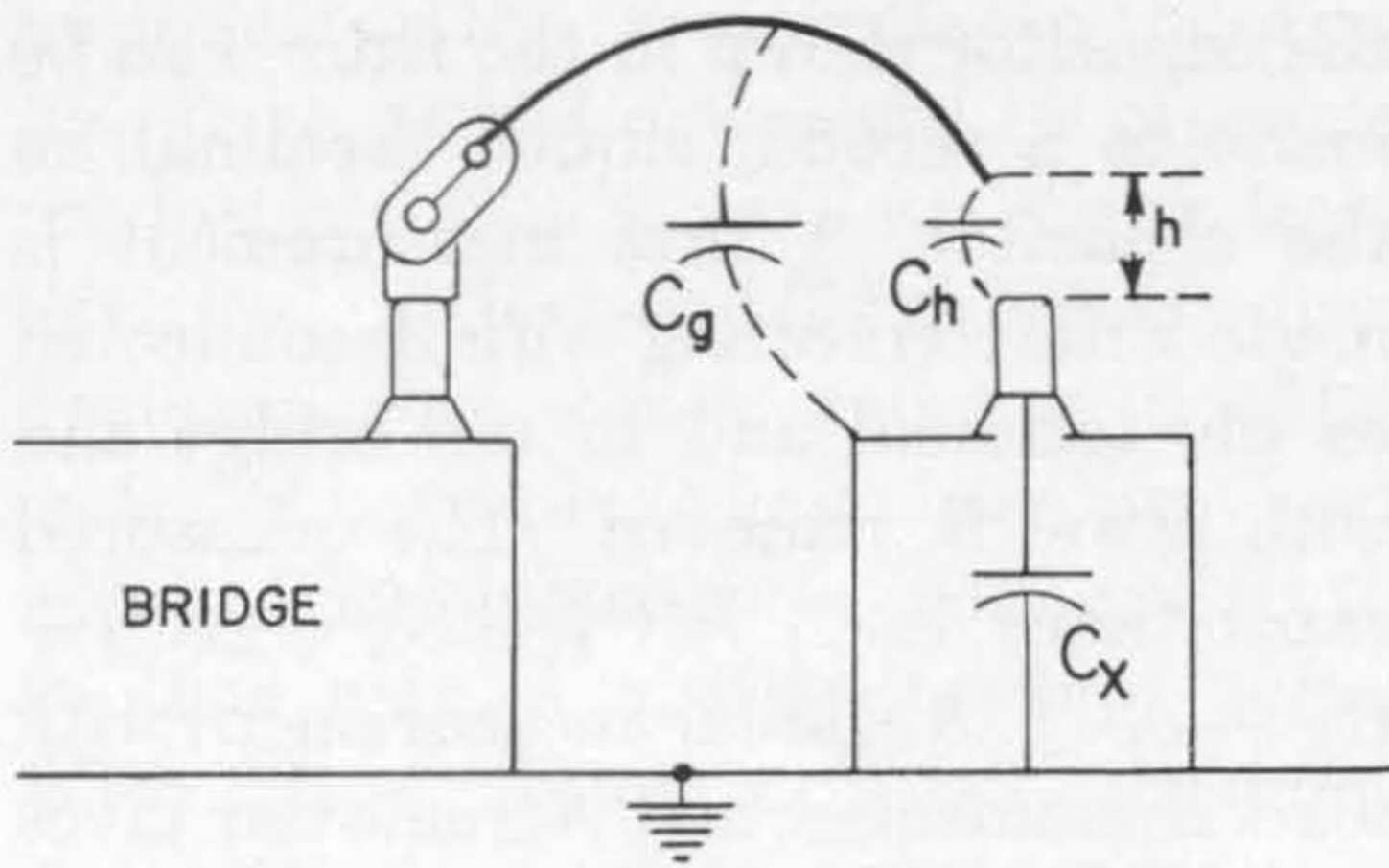


Figure 5. Diagram of connection method using a fine wire.

is difficult to approximate this condition to better than 0.01 pf unless, as in the Rosa and Dorsey capacitor of concentric spheres, one of the conductors surrounds and shields the other.

Another method which removes most, but not all, of the effects of the connections is the fine-wire-connection method described by R. F. Field.⁷ In this method, as shown in Figure 5, the connection between the bridge and the high terminal of the unknown capacitor is made by a wire of small diameter pivoted near the bridge terminal so that its separation from the capacitor terminal can be varied. An initial measurement is made with the wire separated from the terminal by a distance h . With the assumption that the capacitance C_h between the wire and terminal is small compared to the unknown C_x , so that $C_h C_x / (C_h + C_x) \simeq C_h$, the measured capacitance is $C_1 = C_g^h + C_h$, where C_g^h is the capacitance between wire and ground when the separation is h . When the wire is moved in to touch the terminal, the capacitance C_h becomes infinite and the capacitance between wire and ground increases to C_g^o . The measured capacitance now becomes $C_2 = C_g^o + C_x$. The unknown capacitance can thus be related to the measured values C_1 and C_2

⁷R. F. Field, "Connection Errors in Capacitance Measurements," *General Radio Experimenter*, Vol. 12, No. 8, January, 1938, pp. 1-4; reprinted: Vol. 21, No. 11, April, 1947, pp. 1-4.

by $\Delta C = C_2 - C_1 = C_x + (C_g^o - C_g^h - C_h)$. At some particular distance h , the capacitance C_h is equal to the change in C_g as the wire is moved, i.e., $C_h = C_g^o - C_g^h$, and the term in parentheses vanishes, leaving simply $C_x = C_2 - C_1$.

When the wire is curved and pivoted at the bridge end to approach the capacitor terminal from above in the manner described by Field, the change of ΔC with h is fairly linear, as shown in Figure 6 from Field's article. A plot of ΔC against h can then be extrapolated to $h = 0$, where $\Delta C = C_x$, and the value of h which corresponds to this ΔC was found by Field to be $\frac{1}{4}$ inch. The difference between the two capacitances measured with the wire touching the terminal and then $\frac{1}{4}$ inch above it should, therefore, be the value C_x of the unknown capacitor.

This method is simple and useful for calibrations where uncertainties less than 0.1 or 0.2 pf are not significant. It has in the past been used in the calibration of our TYPE 722-D Precision Capacitors, where the direct-reading accuracy limit is $\pm 0.1\%$ or ± 0.2 pf,⁸ and in the calibration of TYPE 1401 Standard Air Capacitors, with a limit of $\pm (0.1\% + 0.1$ pf). There are, however, several reasons why this method has connection errors which can be of the order of 0.1 pf. In the first place, even in the absence of other errors, the capacitance measured, C_x , is not the "free" capacitance (C_x^F), but the capacitance (C_x^w) in the presence of the connecting wire.⁹ Even though the added capacitance to ground of the wire has been eliminated from the measurement, the wire still reduces the capacitance of the terminal

⁸Note that the catalog accuracy limit of ± 0.05 pf for the TYPE 722-ME Precision Capacitor refers to capacitance removed, that is, to a calibration with the capacitance varied but the connections unchanged.

⁹This was pointed out to us by Dr. F. R. Kotter of the National Bureau of Standards.



from its "free" value by disturbing the field around the terminal and, hence, the charge distribution on it. The fine-wire connector used by Field makes the measured capacitance of the TYPE 938 Binding Posts on TYPES 1401 and 722 Capacitors about 0.1 pf less than the "free" capacitance, i.e., $C_x^F = C_x^w + 0.1$ pf.

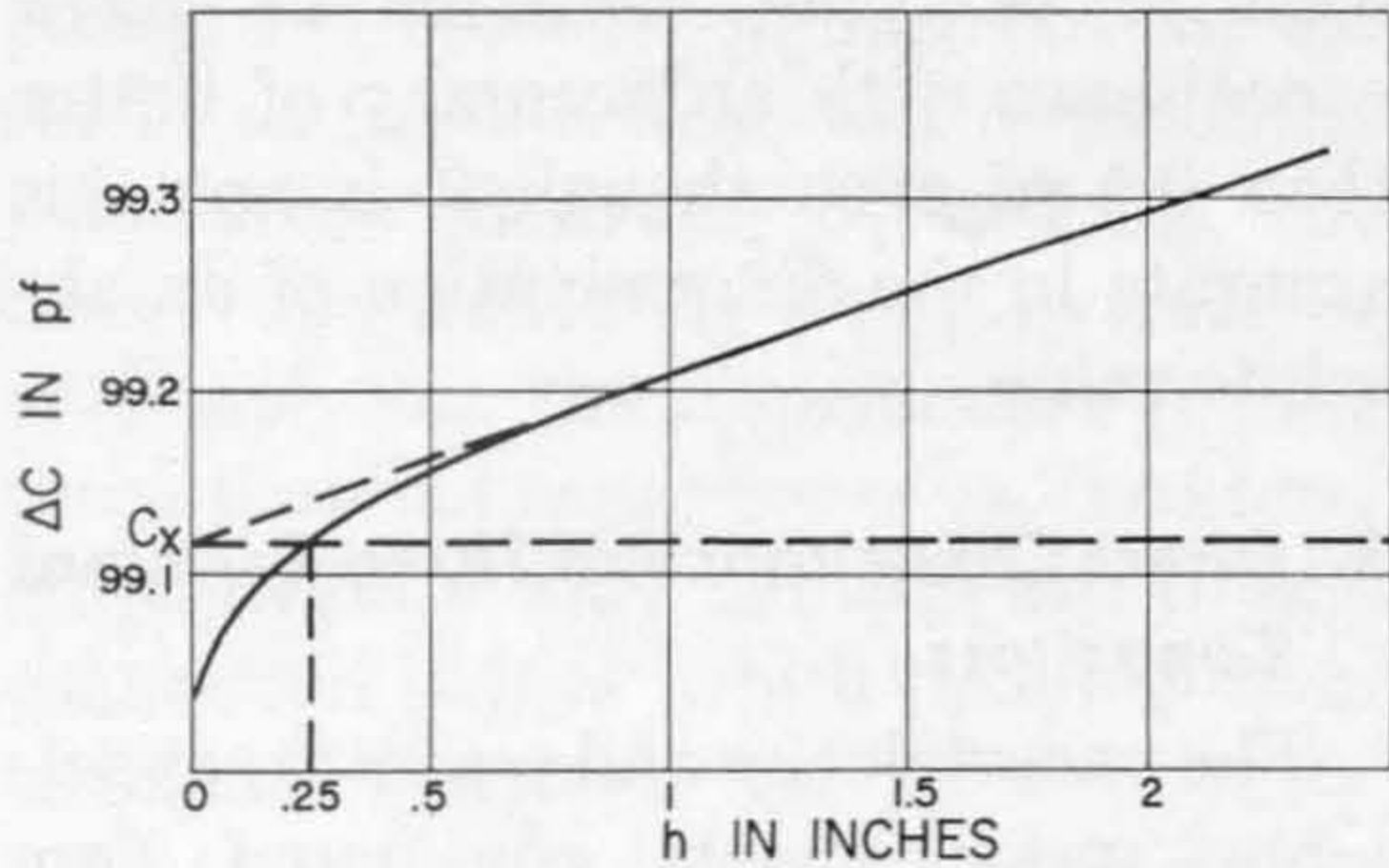


Figure 6. Determination of unknown capacitance C_x from ΔC measured with fine-wire connector. At $h = \frac{1}{4}$ inch $C_x = \Delta C$. (From Field paper⁷)

A recent analysis of the component capacitances in the fine-wire method with a three-terminal capacitance bridge has revealed that even the capacitance C_x^w in the presence of the wire is hard to determine without errors of the order of 0.1 pf. The measurements were made on a capacitor which was externally identical to a TYPE 722-D Precision Capacitor but which had internal, guarded con-

nections to the terminals to permit measurement of the "free" capacitance and the effects of the connector upon it.¹⁰ To simulate the stray capacitances to external grounds in two-terminal measurements, the three-terminal measurements were made with a wire cage (30" x 30" x 30") surrounding the capacitor and its connections and connected to the "ground" terminal of the capacitor.

The measured capacitances, C_x , C_g^h , $C_g^h + C_h$, and $\Delta C = C_x + C_g^o - (C_g^h + C_h)$, are plotted in Figure 7 as a function of the separation h between wire and terminal. For convenience, the capacitance level has been adjusted to make $C_x^F = 100$ pf. The upper curve shows the variation in C_x as the wire moves away from the terminal, with an increase of about 0.1 pf as the influence of the wire vanishes with increasing h . The next curve shows the results of the fine-wire method, with ΔC plotted as a function of h . The lower curves, with the level of the capacitance axis shifted, show the variations of the wire capacitances, C_g^h and $C_g^h + C_h$, with h .

The fine-wire method, when corrected for the effects of the wire on C_x , pre-

¹⁰These measurements were checked within ± 0.01 pf by a measurement of "free" capacitance by the Rosa and Dorsey method.

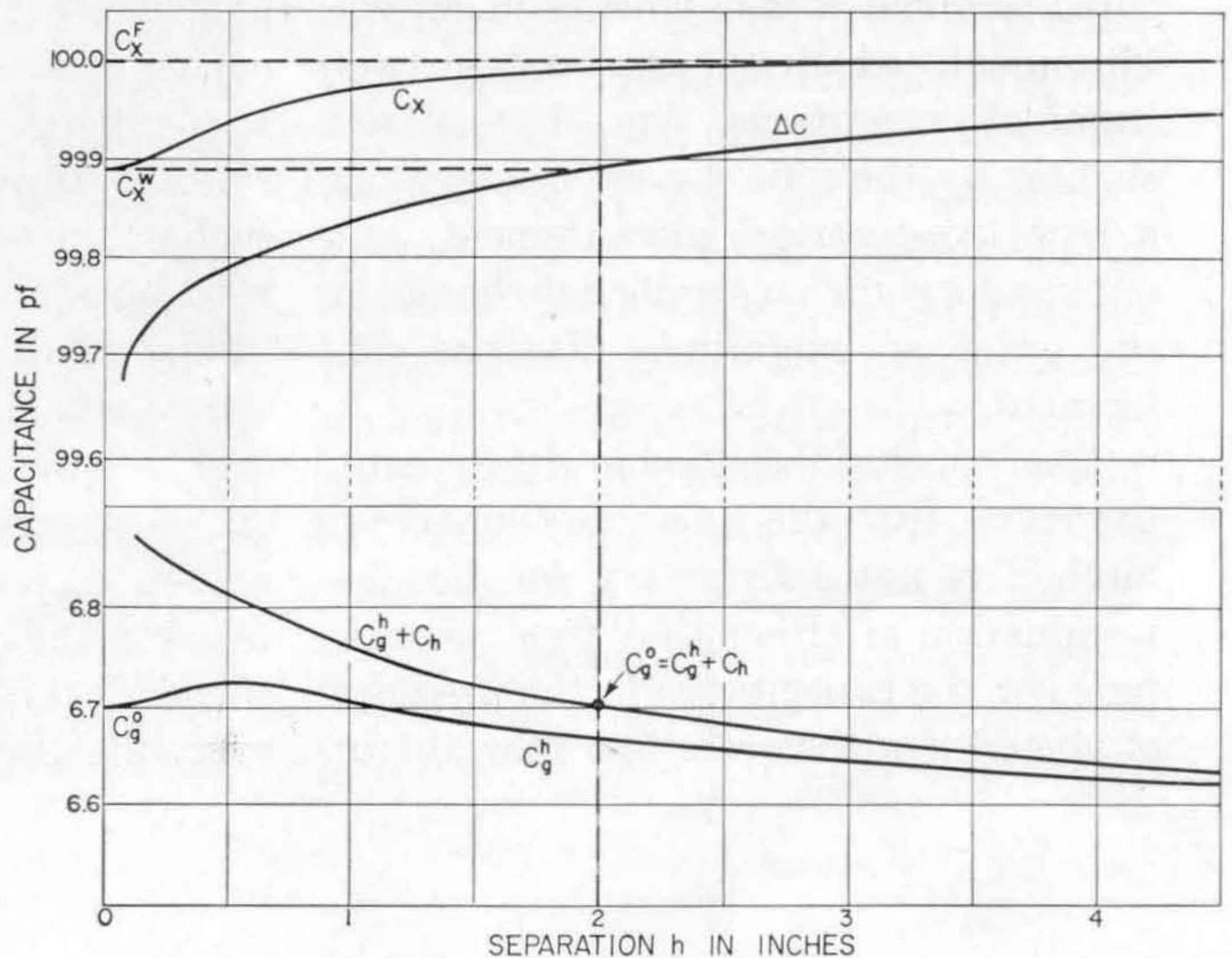


Figure 7. Variation of measured capacitances with fine-wire connector as a function of the separation h .

dicts that, at the distance which makes $C_g^h + C_h = C_g^o$, the capacitance difference ΔC will be equal to C_x^w . In Figure 7 this condition is shown to be satisfied at a distance $h = 2$ inches. Any attempt, however, to determine C_x^w here by extrapolation of the ΔC curve to $h = 0$ encounters difficulty because the curve is not very linear. Examination of the C_g^h , $C_g^h + C_h$, and C_x curves shows that C_h does, as expected, cause deviation from linearity at small h , that C_g^h is itself not linear for h less than about 2.5 inches, and that C_x is not constant with h , as assumed in the derivation of the method. In the region beyond 2.5 inches C_x is almost constant and C_g relatively linear, as desired, and it seems possible that a linear region beyond the range of the graph might extrapolate to C_x^w at $h = 0$.

The important point, however, is the obvious difficulty in determining C_x^w by this method without variations of 0.1 pf. Attention should also be directed to the difference between the ΔC value at the $\frac{1}{4}$ -inch separation and the values of C_x^w and C_x^F . With the $\frac{1}{4}$ -inch spacing, the fine-wire method would give under these conditions a capacitance 0.14 pf less than the capacitance C_x^w in the presence of the wire and 0.25 pf less than the "free" capacitance C_x^F . These results of three-terminal measurements in an environment which approximates two-terminal conditions can be altered slightly by the differing environment in a true two-terminal measurement, but our results show no significant change in the order of magnitude of these differences.

The conclusions to be drawn are, therefore, that the fine-wire-connection method is not satisfactory for the determination of either the "free" capacitance or the capacitance in the presence of the wire with errors less than 0.1 or

0.2 pf unless considerable care is taken in both making and correcting the measurements. As in the other methods of making connection to two-terminal capacitors, the results are reproducible with a precision of 0.01 or 0.02 pf if adequate care is taken to keep the geometry of the connections invariant. The fine-wire method can, therefore, be used to calibrate in terms of *added* capacitance with an accuracy of better than 0.1 pf even though it is not this accurate in the determination of an absolute value.

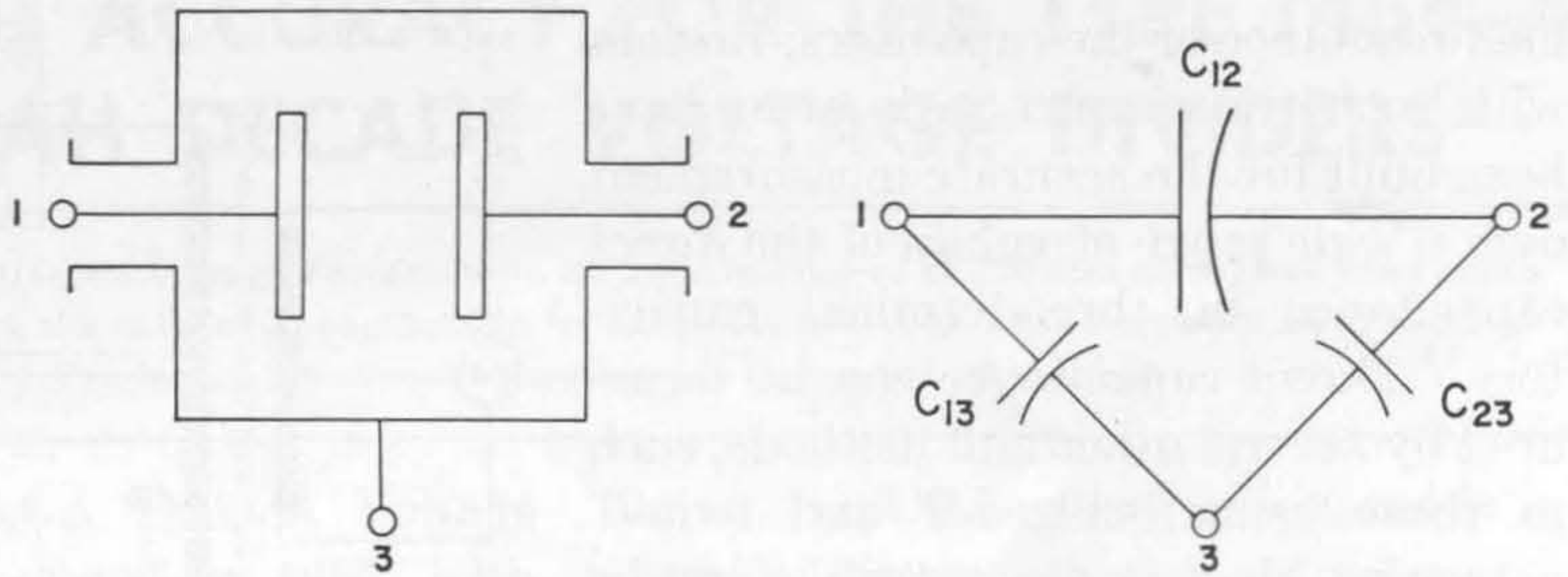
3. Direct Capacitance of Three-Terminal Capacitors.

The uncertainties and errors in capacitance measurements considered here have been the result of the variations in terminal capacitances produced by changes in the connections and in the environment. These problems associated with the capacitor terminals can be eliminated if the terminal capacitances can be separated from the capacitance to be defined and measured. One way of doing this is to introduce a third conductor as a shield or guard which completely surrounds all of at least one of the pair of conductors forming the capacitor to be measured except the area which produces the desired direct capacitance. The pair of conductors of the original capacitors and the added shield form a *three-terminal capacitor*, such as the one shown in Figure 8, along with its equivalent circuit.

The addition of the shield changes the capacitance that existed between 1 and 2 before the introduction of the shield by altering the field, and it also results in new capacitances, C_{13} and C_{23} , between the original conductors and the shield. If the shielding is complete, however, the capacitance C_{12} is now inde-



Figure 8. Diagram and schematic of 3-terminal capacitor.



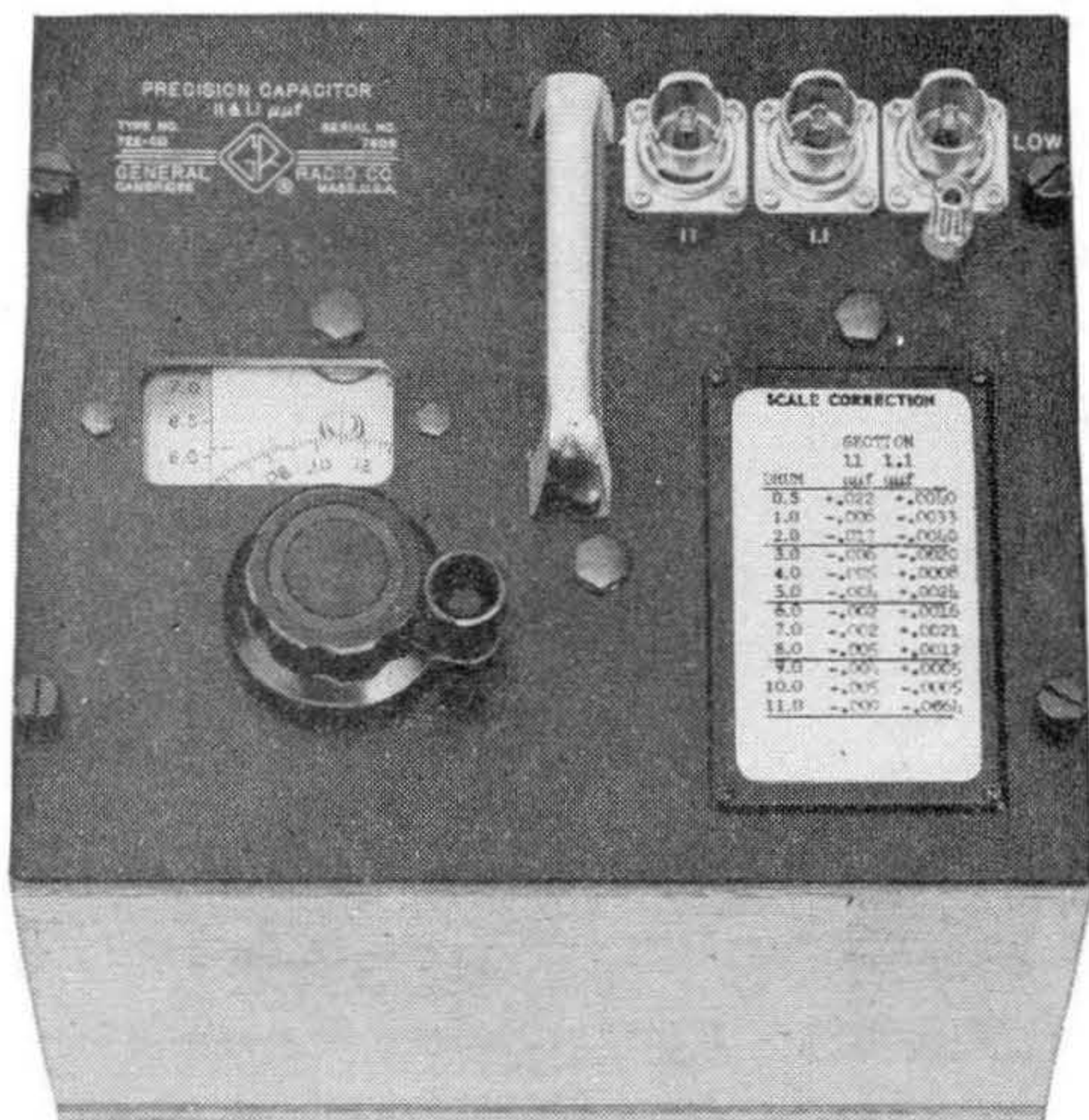
pendent of the surroundings outside the shield and connections to the terminals 1 and 2 can affect only C_{13} and C_{23} . The direct capacitance C_{12} — usually referred to simply as the capacitance of the three-terminal capacitor — is, therefore, quite definite and not subject to the connection errors which trouble two-terminal capacitors.

Note, however, that if in this three-terminal capacitor one of the capacitor terminals, say 2, is connected to the shield, 3, the capacitor reverts to the usual *two-terminal capacitor*, with terminals 1 and 2. The capacitance C_{23} has thus been shorted; the capacitance C_{13} is now parallel with C_{12} , and the capacitor has the capacitance $C = C_{13} + C_{12}$. The

capacitor is, thus, equivalent to that shown in Figure 1. Although C_{12} is still shielded from external influences, C_{13} is a function of connections and environment, and the total capacitance measured is subject to the variations described previously for such two-terminal capacitors.

The well-defined direct capacitance of the three-terminal capacitor is of practical use in a capacitance standard only if it can be measured with high accuracy and with reasonable ease. A bridge with transformer ratio arms is well-suited for just such measurements.¹¹ In the transformer bridge shown in Figure 10, the unknown and standard capacitors are driven by emf's of opposite phase and known ratio from a tapped transformer secondary winding, and the difference in the capacitor currents is measured by a detector. When the bridge is balanced for zero current through the detector, the currents through the direct capacitances C_{12} and C_s must be equal, and the balance relation is $C_{12}/C_s = n$. Any capacitance, such as C_{23} , across the detector has no effect at balance because there is no potential across it. Any capacitance, such as C_{13} , across the transformer winding will have negligible effect on the emf as long as the output impedance of the transformer is small compared to the

Figure 9. View of the Type 722-CD Three-Terminal Precision Capacitor. Outer conductors of Type 874 Coaxial Connectors provide the shield. Inner conductors are the capacitor terminals.



¹¹The three-terminal capacitor and its measurement are well described by A. M. Thompson, "The Precise Measurement of Small Capacitances," I. R. E. Transactions on Instrumentation, Vols. 1-7, Nos. 3 and 4, Dec., 1958, pp. 245-253.

load reactance of the capacitors. Bridges with such transformer ratio arms have been built for the accurate measurement over a wide range of values of the direct capacitance of three-terminal capacitors.¹² Direct capacitance can be measured by several other null methods, such as those using bridged-T and twin-T networks. Most bridge networks can be adapted to the three-terminal measurement by the use of auxiliary bridge arms to balance the unwanted components,¹³ but the double balance required is never convenient and it is difficult to obtain accuracy when the direct capacitance is very small compared to the other capacitances. In some bridges three-terminal measurements can also be made over a limited range by connecting the unwanted capacitances across low-impedance arms of the bridge and across the generator or detector where the shunting effect is negligible.¹⁴

In all these three-terminal measurement methods the connection errors in capacitance can be eliminated by having

¹²The TYPE 1613-A Capacitance Bridge is a transformer bridge covering the range from 5 to 11,000 pf.

¹³The TYPE 716-P4 Guard Circuit provides the components required to make three-terminal measurement with the TYPE 716-C Capacitance Bridge.

¹⁴Three-terminal measurements can be made with the TYPE 1650-A Impedance Bridge in this way.

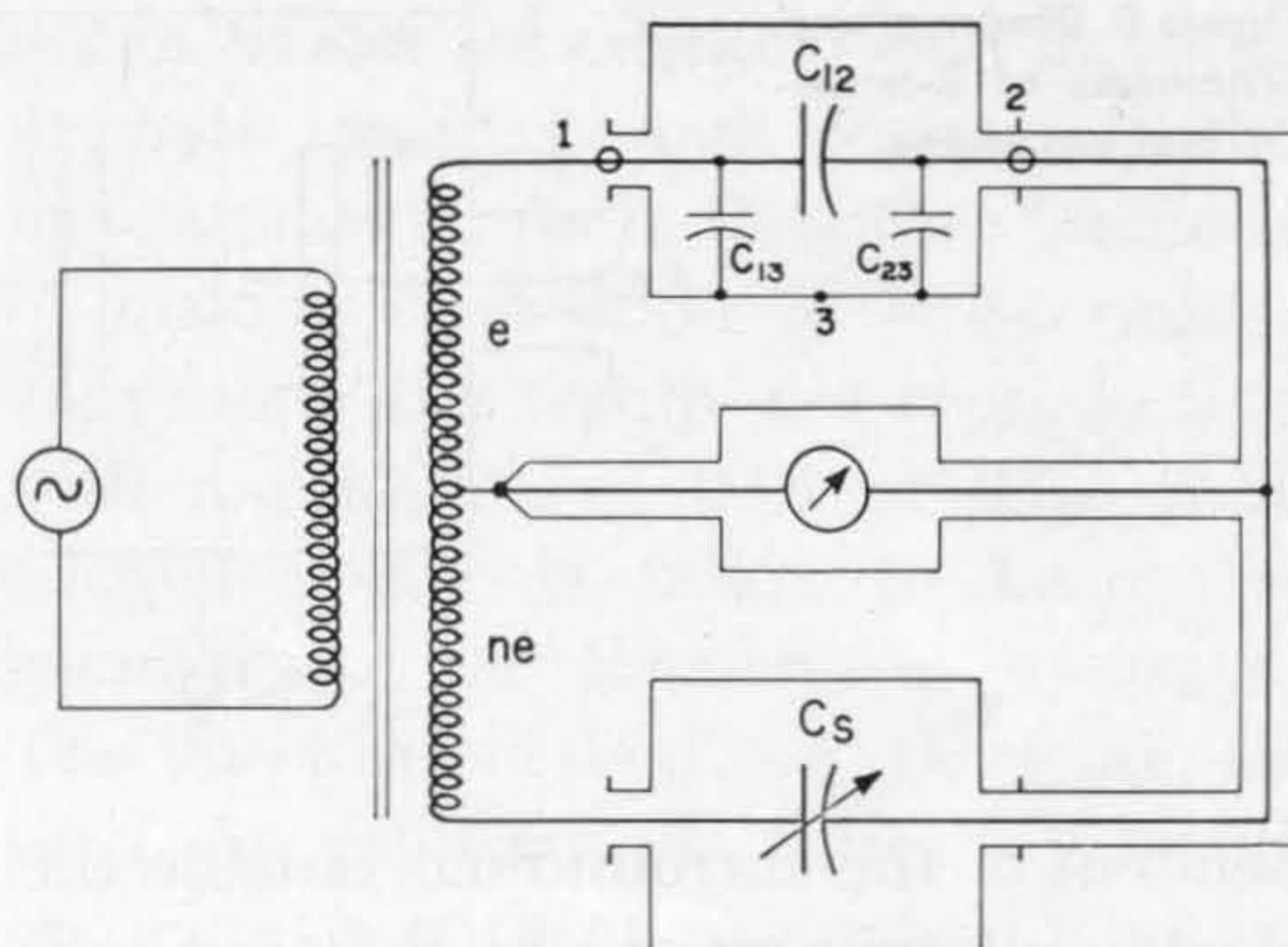


Figure 10. Schematic of transformer-ratio-arm bridge with 3-terminal capacitor connected for measurement.

a complete external shield around at least one of the capacitor terminals. In Figure 10 the case of the capacitor and a shield lead from terminal 2 to the shielded detector complete the shielding around that terminal of the capacitor. A shielded lead to the other terminal is not usually required to eliminate connection capacitances but may be needed to prevent pickup from other sources. With such three-terminal measurements the measured direct capacitance should depend solely upon the construction of the capacitor and the accuracy of measurement should be limited only by the bridge or the reference standards used.

— JOHN F. HERSH

Next month new 3-terminal capacitors, both fixed and variable, will be described. In a forthcoming issue Dr. Hersh will continue the discussion of accuracy considerations and calibration methods for capacitance.

VACATION CLOSING

During the weeks of July 27 and August 3, our Manufacturing Departments will be closed for vacation.

There will be business as usual in the Sales Engineering and Commercial Departments. Inquiries, including requests for technical and commercial informa-

tion, will receive our usual prompt attention.

Our Service Department requests that, because of absences in the manufacturing and repair groups, shipments of equipment to be repaired be scheduled to reach us after the vacation period.